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MATHEMATICAL MODEL FOR SELECTING PRIORITY ENERGY-SAVING MEASURES UNDER UNCERTAINTY AND LIMITED RESOURCES

This article develops a mathematical model for optimizing an investment portfolio of energy-saving measures under fuzzy input parameters and a strictly limited budget. Investment costs and annual energy savings for each candidate measure are modeled as triangular fuzzy numbers and converted into crisp point estimates by combining alpha-cut representations with the Hurwicz criterion asymmetrically: costs tend toward the upper bound of the alpha-cut (reflecting a cautious estimation), while savings tend toward the lower bound. The economic effect of each individual measure is evaluated through its net present value (NPV), computed over the asset's useful service life at a constant discount rate. An integral priority coefficient combines two core components—the per-cost discounted return and the longevity factor—multiplied by a risk-adjustment factor that reflects the decision-maker's specific level of risk aversion. The selection task is formulated as a 0–1 knapsack problem and solved via dynamic programming. The robustness of the generated optimal plan is systematically assessed using a vectorized Monte Carlo simulation of 8,000 realizations, with subsequent calculation of the 5%, 50%, and 95% quantiles. The model's real-world applicability is demonstrated through a practical case study of a healthcare complex with ten candidate measures. Ultimately, the model provides a transparent ranking of measures, a feasible investment plan within the prescribed budget, and an interval-valued NPV estimate, thereby supporting efficient investment decisions under uncertainty.

Keywords: energy saving, mathematical model, triangular fuzzy numbers, net present value, knapsack problem, Monte Carlo, healthcare.

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МАТЕМАТИЧНА МОДЕЛЬ ВИБОРУ ПРІОРИТЕТНИХ ЗАХОДІВ ЕНЕРГОЗБЕРЕЖЕННЯ ЗА УМОВ НЕВИЗНАЧЕНОСТІ І ОБМЕЖЕНОСТІ РЕСУРСІВ

У статті побудовано математичну модель формування інвестиційного портфеля заходів енергозбереження за умов нечіткості вхідних параметрів та обмеженого бюджету. Інвестиційні витрати й прогнозовану економію подано трикутними нечіткими числами, дефазифіковано через α -зріз і асиметричний критерій Гурвіца. Економічну ефективність оцінено через чисту приведену вартість (NPV). Інтегральний коефіцієнт пріоритетності поєднує питому NPV-віддачу і фактор довговічності, скориговані на ризик впровадження. Задачу вибору сформульовано як 0–1 ранцеву та розв'язано методом динамічного програмування. Стійкість плану перевірено векторизованим методом Монте-Карло з оцінкою квантилів P5, P50, P95. Працездатність моделі показано на прикладі десяти типових заходів для лікарняного комплексу.

Ключові слова: енергозбереження, математична модель, трикутні нечіткі числа, NPV, ранцева задача, Монте-Карло, охорона здоров'я.

Problem statement. The armed aggression against Ukraine has substantially intensified the issue of rational use of fuel and energy resources by all categories of consumers, especially budget-funded and municipal



enterprises – healthcare facilities, schools, water-supply and district-heating utilities. In most cases the manager of such an object simultaneously faces several interconnected problems: an extensive yet limited list of technically feasible energy-saving measures; a strictly fixed investment budget; uncertainty regarding both the cost of works and the expected energy savings (in particular due to market volatility and dependence of the effect on the building operating regime); the need to account for the risk of incomplete implementation or under-performance of design effectiveness. Under these conditions, classical deterministic payback analysis is insufficient: it reflects neither the interval nature of input data nor the long-term behaviour of the aggregate cash flow. It is therefore timely to construct a mathematical model that simultaneously formalises the uncertainty of input parameters and the limitedness of resources, while reconciling the priorities of measures by discounted financial return and risk; such an approach must align with updated international guidance on resilient and energy-efficient healthcare facilities [13; 14] and with the European regulatory framework on energy efficiency [15].

Analysis of recent studies and publications. The authors of the present paper have previously developed a hybrid neural-network apparatus for assessing the intellectual component of the energy-saving policy of an enterprise [1] and an approach to modelling the potential of industrial enterprises under uncertainty [2]. Modern economic mechanisms for managing energy saving are systematised in the collective monograph [3].

The classical theory of fuzzy sets was introduced in the seminal publication by L. Zadeh [8]; the formal apparatus of triangular fuzzy numbers and operations with α -cuts is systematically presented in [9; 10]. A recent (2023) review of methods for multi-criteria decisions with fuzzy input parameters together with an analysis of MCDA application trends is given in [11; 12]. The theoretical basis of the Hurwicz criterion, underlying the asymmetric defuzzification proposed here, is given in [4]. The classical monograph on the 0–1 knapsack problem is presented in [5], the methodology for solving it by dynamic programming in [6], and a standard reference on Monte Carlo methods in financial analysis is [7].

The sectoral context of energy saving in healthcare facilities is defined by the WHO Operational Framework for Building Climate Resilient and Low Carbon Health Systems [13] and by the IEA Energy Efficiency 2024 report [14]; at the EU level the policy is regulated by Directive (EU) 2023/1791 [15].

A combined mathematical model that simultaneously integrates triangular fuzzy numbers, asymmetric defuzzification by the Hurwicz criterion, discounted cash flow, the 0–1 knapsack problem and a vectorised Monte Carlo method specifically for the context of healthcare facilities is, however, insufficiently represented in the literature, which motivates further research.

Purpose and objectives of the study. The aim of the paper is to construct a mathematical model – transparent for practical application – for selecting priority energy-saving measures under fuzzy input data and limited investment resources. The research objectives are: to formalise the description of investment costs and annual savings as triangular fuzzy numbers; to propose an asymmetric defuzzification algorithm based on the Hurwicz criterion; to introduce an integral priority coefficient with an NPV component and a risk adjustment; to formulate the selection task as a 0–1 knapsack problem and solve it by dynamic programming; to implement a vectorised Monte Carlo method for quantile estimation; and to demonstrate the model on a case study of a healthcare complex.

Presentation of the main research material. For each i -th energy-saving measure the model operates on two key parameters: investment costs C_i and annual electricity savings S_i . In design practice both quantities are known only approximately: cost depends on suppliers' quotations and market fluctuations, while the expected savings depend on weather conditions, the building operating schedule and the quality of works. The most convenient form for describing such uncertainty [8] is the triangular fuzzy number [9; 10] defined by a triple (a_i, b_i, c_i) , where a_i is the optimistic (minimum) estimate, b_i – the most likely value, and c_i – the pessimistic (maximum) estimate. This format matches the natural manner of expert wording, does not require a full probability distribution and integrates conveniently with matrix calculations.

To convert the triangular number into a point estimate suitable for the optimisation task, the model employs a two-step defuzzification procedure: first the alpha-cut at confidence level α within $[0, 1]$ is computed (the interval representation of the fuzzy number); then this interval is folded into a point value by the Hurwicz optimism criterion [4]. Both steps are combined into a single expression:

$$V(\tilde{A}; \alpha, h) = (1 - h)(a + \alpha(b - a)) + h(c - \alpha(c - b)) \quad (1)$$

where $\tilde{A} = (a, b, c)$ is the triangular fuzzy number describing the uncertain parameter; a – optimistic estimate; b – most likely value; c – pessimistic estimate; α within $[0, 1]$ – confidence level for the alpha-cut ($\alpha = 0.7$ in calculations); h within $[0, 1]$ – optimism parameter in the Hurwicz convolution ($h = 0$ fully pessimistic, $h = 1$ fully optimistic); V – resulting point value of the parameter.

We note the boundary behaviour of formula (1). At $\alpha = 0$ the alpha-cut coincides with the support of the fuzzy number $[a, c]$, so the Hurwicz convolution gives $V = (1 - h) \cdot a + h \cdot c$. At $\alpha = 1$ the interval degenerates



to the single point b , hence $V = b$ regardless of h : all information about uncertainty is lost. The extreme values of α are therefore not used in practice. The model adopts an intermediate value $\alpha = 0.7$, which corresponds to a moderately high level of expert confidence: the width of the resulting interval equals 30% of the support width, which is sufficient to preserve the uncertainty information without exceeding the limits of reasonable expert estimation.

A methodologically essential feature of the model, consistent with the authors' prior fuzzy modelling methodology [2], is the asymmetric choice of the optimism parameter h for the two categories of inputs. For investment costs we adopt $h_c = 0.6$ – gravitating toward the upper bound of the interval (a larger, cautious estimate of cost); for the expected annual savings – $h_s = 0.4$ – gravitating toward the lower bound (a smaller, cautious estimate of effect). This asymmetry protects the model from the classical optimism bias of deterministic payback calculations, where both cost and effect are assessed too optimistically. The numerical values $h_c = 0.6$ and $h_s = 0.4$ (a symmetric deviation of ± 0.1 from the neutral value $h = 0.5$) are chosen as moderately conservative: they provide a 10% margin of caution on each side without an excessive pessimism that would devalue the majority of measures at the screening stage. In the sensitivity analysis the user of the model may vary h_c , h_s within $[0.5; 0.7]$ in order to verify the robustness of the result.

$$C_i = (1 - h_c)(a_i^c + \alpha(b_i^c - a_i^c)) + h_c(c_i^c - \alpha(c_i^c - b_i^c)), \quad h_c = 0.6 \quad (2)$$

where C_i – defuzzified investment costs of the i -th measure, thousand UAH; a_i^c , b_i^c , c_i^c – parameters of the triangular fuzzy number for cost; $h_c = 0.6$ – optimism parameter for cost; α – confidence level as in formula (1).

$$S_i = (1 - h_s)(a_i^s + \alpha(b_i^s - a_i^s)) + h_s(c_i^s - \alpha(c_i^s - b_i^s)), \quad h_s = 0.4 \quad (3)$$

where S_i – defuzzified annual energy savings in natural units, thousand kWh per year; a_i^s , b_i^s , c_i^s – parameters of the triangular fuzzy number for savings; $h_s = 0.4$ – optimism parameter for savings.

For further economic calculations the natural savings are converted to a monetary equivalent by multiplying by the prevailing tariff:

$$E_i = S_i \cdot \tau \quad (4)$$

where E_i – annual savings of the i -th measure in monetary terms, thousand UAH; S_i – defuzzified annual savings, thousand kWh per year; τ – prevailing electricity tariff, UAH per kWh ($\tau = 5.20$ UAH per kWh).

The economic effect of each measure is evaluated through the classical net present value (NPV) indicator. Future cash inflows are discounted to the present by rate r , reflecting the opportunity cost of capital for a budget-funded institution. The calculations assume $r = 12$ percent, corresponding to the weighted-average cost of capital for public-sector projects under elevated inflation. The discount period is limited by the useful service life T_i of each measure within the overall planning horizon $H = 10$ years.

$$NPV_i = \sum_{t=1}^{T_i} \frac{E_i}{(1+r)^t} - C_i \quad (5)$$

where NPV_i – net present value of the i -th measure, thousand UAH; E_i – annual monetary savings from formula (4); C_i – investment costs from formula (2); r – discount rate ($r = 0.12$); t – running year; T_i – useful service life (limited by horizon H). A positive NPV_i means the measure delivers discounted profit exceeding the initial investment.

To reduce all economic characteristics of a measure to a single quantitative measure suitable for the optimisation task, the model introduces the integral priority coefficient K_i . It combines two key components: the per-cost NPV efficiency (how many UAH of discounted profit each invested UAH yields) and the longevity factor (measures that more rapidly release invested capital are considered more attractive in portfolio dynamics). The resulting value is reduced by a risk adjustment: the higher the expert risk index r_i and the more conservatively the decision-maker is configured (larger λ), the stronger the penalty:

$$K_i = \left(w_1 \cdot \frac{NPV_i}{C_i} + w_2 \cdot \frac{1}{T_i} \right) \cdot (1 - \lambda r_i) \quad (6)$$

where K_i – integral priority coefficient of the i -th measure; NPV_i – net present value from formula (5); C_i – investment costs from formula (2); T_i – useful service life, years; w_1 , w_2 – normalised weights, $w_1 + w_2 = 1$ (in calculations (0.70; 0.30)); λ within $[0, 1]$ – risk-aversion coefficient ($\lambda = 0.4$ in base calculations); r_i within $[0, 1]$ – expert estimate of the i -th measure implementation risk. Regarding the dimensionality of the terms in formula (6): the first term $w_1 \cdot (NPV_i / C_i)$ is dimensionless (thousand UAH / thousand UAH). The second term $w_2 \cdot (1/T_i)$ formally has the dimension $[1/\text{year}]$. Within this model the quantity $1/T_i$ is interpreted as a numerical index of the rate of return of invested capital over the years of useful life, with a conventional dimensionlessness, since it is used exclusively for ranking measures within the same planning horizon and is



not compared with absolute financial values. The weights $(w_1, w_2) = (0.70; 0.30)$ are chosen to give priority to economic return over the longevity factor at the screening stage; the coefficient $\lambda = 0.4$ corresponds to a moderately conservative attitude to risk of the decision-maker in a budget-funded institution.

The selection task is formulated as the classical 0–1 knapsack problem [5]: among n potential measures one must choose a subset that maximises the total integral coefficient subject to the constraint that total investment costs do not exceed the available budget. Each measure is either fully implemented or not implemented at all – hence the binary constraint on decision variables:

$$F(x) = \sum_{i=1}^n K_i \cdot x_i \rightarrow \max \quad (7)$$

where $F(x)$ – objective function (total plan priority); n – total number of potential measures; x_i – binary decision variable for inclusion of the i -th measure; K_i – integral priority coefficient from formula (6); $x = (x_1, \dots, x_n)$ – decision vector.

subject to the budget constraint:

$$\sum_{i=1}^n C_i \cdot x_i \leq B \quad (8)$$

where B – investment budget, thousand UAH ($B = 2\,000$ in calculations); C_i – defuzzified investment costs from formula (2). Constraint (8) forbids exceeding the available financing.

and the binary constraint on decision variables:

$$x_i \in \{0, 1\}, \quad i = \overline{1, n} \quad (9)$$

where $x_i = 1$ – measure is fully implemented; $x_i = 0$ – measure is not implemented. The integrality constraint makes the task combinatorial; it is solved by the classical dynamic-programming method [6] with $O(n \text{ times } B)$ complexity, which for $n = 10$ and $B = 2\,000$ yields approximately 20 000 elementary operations – a fraction of a second of computation.

Once the deterministic optimal plan x^* is found, the natural question is: how stable is the aggregated result under variations of input parameters? The model addresses this via a vectorised Monte Carlo simulation [7]. On each of N realisations ($j = \overline{1, \dots, N}$) the cost $\tilde{C}_i^{(j)}$ and natural savings $\tilde{S}_i^{(j)}$ of each selected measure are independently drawn from the corresponding triangular distributions. The NPV of the aggregated cash flow of the selected plan on the j -th realisation is computed by the expression:

$$NPV^{(j)} = \sum_{i: x_i^* = 1} \left[\tilde{S}_i^{(j)} \cdot \tau \cdot \sum_{t=1}^{T_i} \frac{1}{(1+r)^t} \right] - \sum_{i: x_i^* = 1} \tilde{C}_i^{(j)} \quad (10)$$

where $NPV^{(j)}$ – realisation of NPV on the j -th Monte Carlo sample; $x^* = (x_1^*, \dots, x_n^*)$ – optimal plan; $\tilde{S}_i^{(j)}$, $\tilde{C}_i^{(j)}$ – realisations of the triangular distribution for savings and costs of the i -th measure on the j -th sample; τ , r , T_i – parameters from previous formulas. The set $\{NPV^{(j)}\}$ for j from 1 to N forms the empirical distribution from which quantile estimates P_s , P_{50} and P_{95} are computed. The calculations use $N = 8\,000$ realisations – sufficient for stable quantile estimates given the present input-parameter spread.

To validate the model, a list of ten typical energy-saving measures relevant to a hospital block at the second level of medical care [13; 14] has been compiled. The case object is an illustrative model example based on the generalised characteristics of a central district hospital in climatic zone I B of Ukraine (mean annual temperature $+7.5^\circ\text{C}$, heating season approximately 187 days); the baseline annual energy consumption used for computing savings is approximately 320 MWh of electrical and 1 100 Gcal of thermal energy. The numerical values of costs and savings for individual measures are derived from average design data and market commercial offers of 2024–2025, which allows the workability of the model to be demonstrated illustratively without binding to a specific physical object. Investment budget $B = 2\,000$ thousand UAH; planning horizon $H = 10$ years; tariff $\tau = 5.20$ UAH per kWh; discount rate $r = 12$ percent; confidence level $\alpha = 0.7$; risk-aversion coefficient $\lambda = 0.4$. Parameters (a_i, b_i, c_i) for cost and savings are based on expert estimation. The input data for the case study – the triples of triangular fuzzy numbers (a, b, c) for the investment costs and annual savings together with the expert estimates of the implementation risk r_i for each of the ten measures – are reported in Table 1; the integral indicators and the optimal selection are reported in Table 2.

Notes to Table 1. Investment costs a^c , b^c , c^c are given in thousand UAH (optimistic, most likely, pessimistic estimates respectively); annual electricity savings in natural units a^s , b^s , c^s are in thousand kWh per year; the expert estimate of the implementation risk r_i is given in dimensionless units in the range $[0; 1]$. The useful service life T_i of each measure is given in Table 2. The defuzzified values C_i and E_i in Table 2 are computed from formulas (2)–(4) with $\alpha = 0.7$, $h_c = 0.6$, $h_s = 0.4$, $\tau = 5.20$ UAH per kWh, fully reproducing the integral indicators of Table 2.



Table 1

**Input data: triples of triangular fuzzy numbers for costs and savings
and expert estimates of the implementation risk**

No.	Measure	a^c	b^c	c^c	a^s	b^s	c^s	r_i
1	LED lighting	111	123	148	11.3	12.9	15.2	0.10
2	External wall insulation	491	546	655	36.3	41.3	49.7	0.25
3	Window replacement	722	802	961	31.1	35.4	42.4	0.20
4	Ventilation heat recovery	409	454	545	24.2	27.5	32.6	0.30
5	Heat pump	998	1109	1333	61.2	69.5	83.6	0.35
6	Solar collectors (DHW)	321	356	429	18.1	20.5	24.7	0.25
7	Boiler modernisation	543	604	722	29.6	33.7	40.4	0.20
8	BMS automation	228	254	302	13.9	15.8	18.9	0.15
9	Pipeline insulation	82.3	91.4	110	9.5	10.7	13.1	0.10
10	Photovoltaic plant	864	960	1152	45.1	51.3	61.5	0.30

Source: authors' expert estimates.

Table 2

Results of computing the integral indicators and the optimal selection of measures

No.	Measure	C, k UAH	E, k UAH	NPV, k UAH	T, yr	K	Selected
1	LED lighting	126.0	67.0	206.7	8	1.138	+
2	External wall insulation	559.0	215.3	657.4	10	0.768	+
3	Window replacement	821.0	184.4	220.9	10	0.201	-
4	Ventilation heat recovery	465.0	143.1	343.6	10	0.482	+
5	Heat pump	1136.0	362.4	911.9	10	0.509	-
6	Solar collectors (DHW)	365.0	107.0	239.7	10	0.441	+
7	Boiler modernisation	618.0	175.6	373.9	10	0.417	-
8	BMS automation	259.6	82.3	205.2	10	0.548	+
9	Pipeline insulation	93.6	56.0	222.5	10	1.627	+
10	Photovoltaic plant	983.0	267.3	527.2	10	0.357	-
	Total of the selected plan	1868.2	670.6	1875.1	–	–	–

Source: authors' calculations.

The optimal plan comprises six measures with a total cost of 1 868.2 thousand UAH (93 percent of the budget); the remaining available budget is 131.8 thousand UAH ($B - \sum C_i \cdot x_i = 2\,000 - 1\,868.2$), which is insufficient to add any of the three excluded measures; the total NPV of the selected plan is 1 875.1 thousand UAH and the total annual savings is 670.6 thousand UAH. Measures with relatively high individual NPV (heat pump, photovoltaic plant, boiler modernisation) did not enter the plan, because under the limited budget their inclusion would displace several cheaper measures with higher K. A graphical ranking of measures by the K coefficient is shown in Fig. 1.

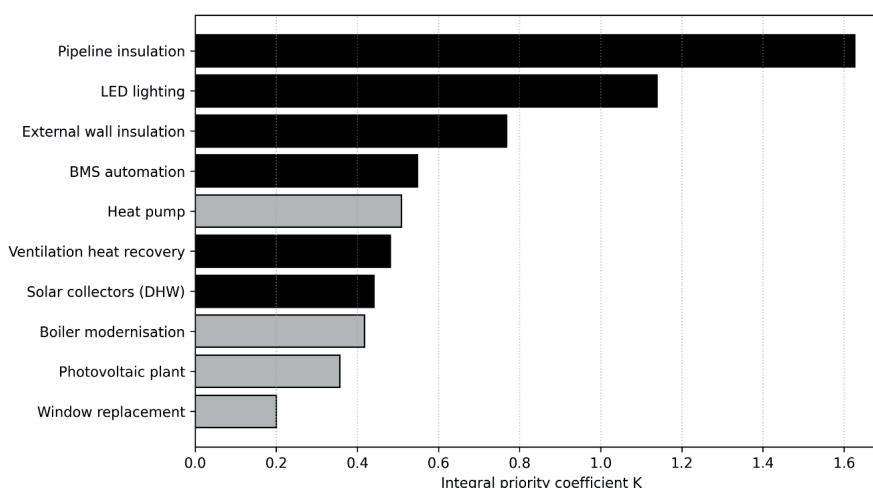


Fig. 1. Ranking of measures by the integral priority coefficient



In Fig. 2 a cost-NPV map is built with horizontal and vertical uncertainty intervals derived from the alpha-cut. Measures located in the upper-left part of the chart (low cost, high NPV) form the core of investment-attractive measures.

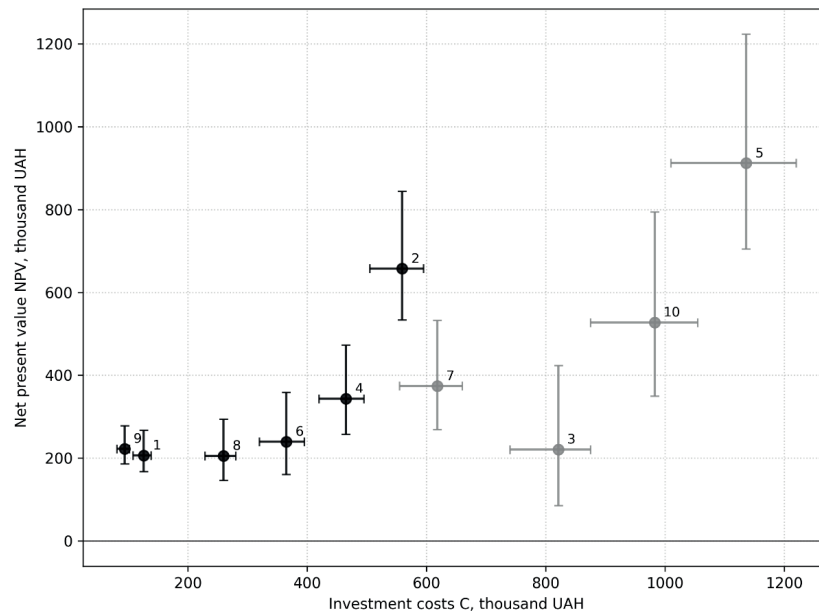


Fig. 2. Cost-NPV map with uncertainty intervals

The Monte Carlo simulation results (Fig. 3) show that the median NPV of the aggregated cash flow of the selected plan equals 1 965 thousand UAH; with probability 90 percent the actual NPV lies in the range from 1 571 to 2 364 thousand UAH. The distribution is relatively narrow (P95 over P5 is approximately 1.5), indicating stability of the aggregated financial outcome under variations of input parameters.

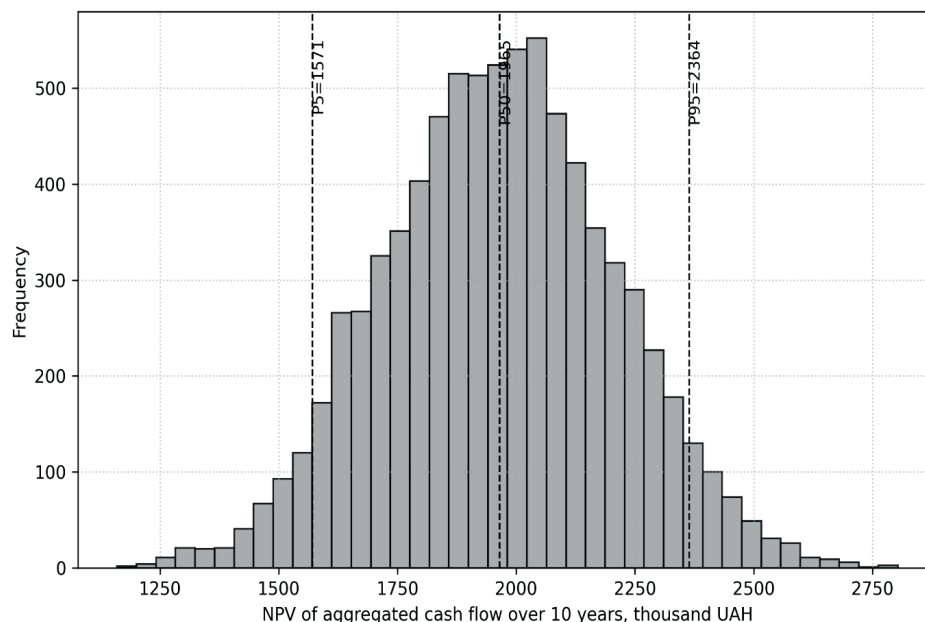


Fig. 3. NPV distribution (Monte Carlo method)

The budget-sensitivity curve (Fig. 4) exhibits a characteristic stepped pattern: each jump corresponds to the inclusion of the next measure. The marginally useful budget for the considered facility is estimated at 2 800–3 200 thousand UAH; beyond this level the growth of annual savings slows considerably.

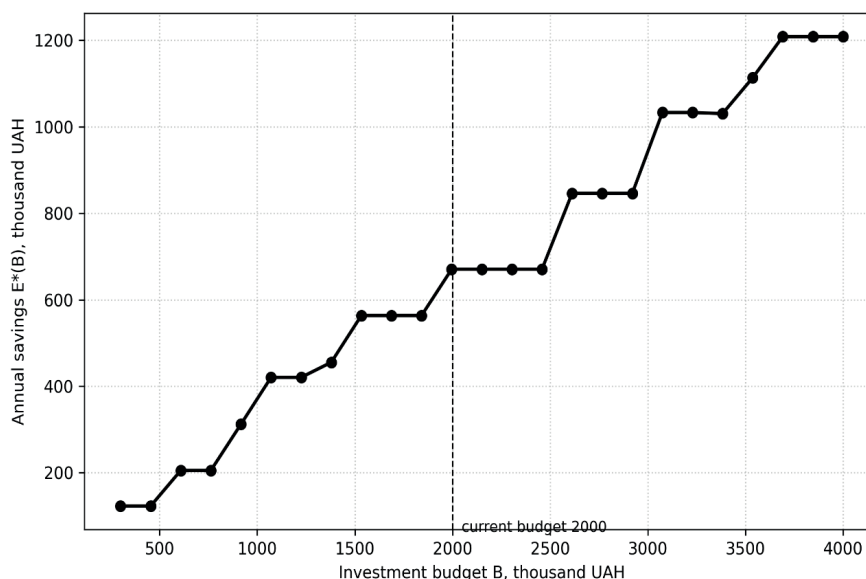


Fig. 4. Annual savings as a function of the investment budget

Conclusions. The proposed simplified mathematical model combines the apparatus of triangular fuzzy numbers, asymmetric defuzzification by the Hurwicz criterion, discounted cash flow, a two-component integral priority coefficient, the 0–1 knapsack problem solved by dynamic programming, and a vectorised Monte Carlo method. This combination provides a formalised and transparent selection of the measure portfolio under simultaneously present uncertainty of input data and limitation of investment resources, while remaining sufficiently simple for practical application. Validation on a hospital complex of ten typical measures shows that the optimal plan contains six measures with a total cost of 1 868 thousand UAH (93 percent of the budget) and delivers a total NPV of 1 875 thousand UAH at a Monte Carlo median of 1 965 thousand UAH and a 5 percent lower quantile of 1 571 thousand UAH, confirming the economic efficiency of the decision. Further research directions include extending the model to multi-period planning with phased measure implementation, incorporating tariff escalation and synergy effects between measures, and integrating the model with state support programs for energy-efficient investment.

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